Appl. Phys. B 69, 327–332 (1999) / Digital Object Identifier (DOI) 10.1007/s003409900148

Applied Physics B
Lasers
and Optics

Carrier-envelope offset phase control: A novel concept for absolute optical frequency measurement and ultrashort pulse generation

H.R. Telle[1], G. Steinmeyer[2], A.E. Dunlop[2], J. Stenger[1], D.H. Sutter[2], U. Keller[2]

[1] Bundesanstalt (PTB), Braunschweig, Germany
(Fax: +49-531/592-4423, harald.telle@ptb.de)
[2] Institut für Quantenelektronik, ETH Zürich, Switzerland
(Fax: +41-1/633-1059, sguenter@iqe.phys.ethz.ch)

Received: 19 August 1999/Published online: 8 September 1999

Abstract. The shortest pulses periodically emitted directly from a mode-locked Ti:sapphire laser are approaching the two-optical-cycle range. In this region, the phase of the optical carrier with respect to the pulse envelope becomes important in nonlinear optical processes such as high-harmonic generation. Because there are no locking mechanisms between envelope and carrier inside a laser, their relative phase offset experiences random fluctuations. Here, we propose several novel methods to measure and to stabilize this carrier-envelope offset (CEO) phase with sub-femtosecond uncertainty. The stabilization methods are an important prerequisite for attosecond pulse generation schemes. Short and highly periodic pulses of a two-cycle laser correspond to an extremely wide frequency comb of equally spaced lines, which can be used for absolute frequency measurements. Using the proposed phase-measurement methods, it will be possible to phase-coherently link any unknown optical frequency within the comb spectrum to a primary microwave standard. Experimental studies using a sub-6-fs Ti:sapphire laser suggesting the feasibility of carrier-envelope phase control are presented.

PACS: 06.30.Ft; 42.65.Re; 42.65.Ky

Recently, light pulses generated directly from a cw mode-locked laser with a duration below 6 fs have been reported [1, 2]. The pulses contain approximately two optical cycles within their intensity half-width. In this regime, the carrier-envelope offset phase ϕ_{CEO} (has an influence on the conversion efficiency of nonlinear optical processes. This is also referred to as the breakdown of the slowly varying envelope approximation (SVEA). Generally, the CEO phase is expected to influence any optical process that depends nonlinearly on the instantaneous electric field strength rather than on the intensity. Measurement and control of ϕ_{CEO} is crucial for applications of high-harmonic generation [3]. In particular, certain attosecond pulse generation schemes require a constant ϕ_{CEO} [4, 5].

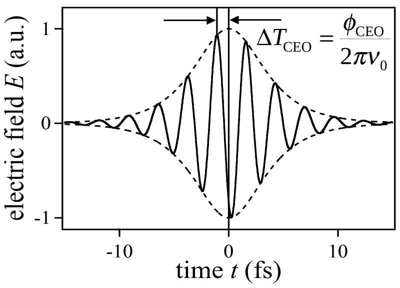

Fig. 1. 5-fs pulse with sech-shaped field envelope. The pulse-to-pulse carrier-envelope offset phase, ϕ_{CEO}, is proportional to the roundtrip delay of the central fringe, ΔT_{CEO}, and the carrier frequency, ν_0. ϕ_{CEO} is defined as the phase angle at pulse center

Apart from the applications in ultrafast optics, tracking of ϕ_{CEO} allows for the synthesis of an optical frequency [6] directly from a microwave frequency standard, such as the primary standard of time and frequency, the Cs atomic clock. A dense grid of reference frequencies throughout substantial parts of the visible to near infrared spectral range can be created. A relative frequency uncertainty of 10^{-14} or lower seems to be possible and this would open new levels of precision in areas such as ultra-high-resolution spectroscopy, optical frequency standards, and the testing of fundamental theories.

In the carrier frequency domain, the modes of a periodic pulse train of a laser are represented by a comb of lines equally spaced by f_{rep}, the pulse repetition frequency (Fig. 2). Recent research shows that the spacing of the longitudinal laser modes of a 73-fs Kerr-lens mode-locked (KLM) Ti:sapphire laser differs from the pulse repetition frequency by less than 10^{-15} and that the frequency spacing of the comb modes between 823 nm and 871 nm is uniformly distributed within an experimental resolution of better than 10^{-16} [7]. These phase-locked modes must not be confused with cavity resonance frequencies which are not equally spaced in the presence of intracavity dispersion.

To discuss ϕ_{CEO} in the frequency domain, it is useful to introduce its time derivative and the group-phase off-

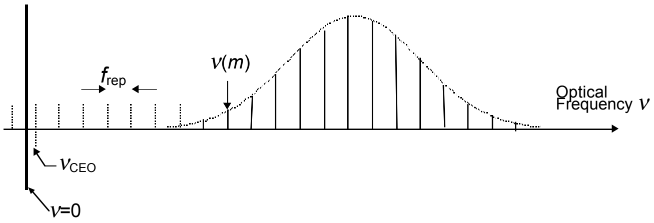

Fig. 2. Representation of carrier-envelope offset frequency ν_{CEO} in the carrier frequency domain

set frequency ν_{GPO}, and to define the carrier-envelope offset frequency ν_{CEO} from ν_{GPO} as

$$\nu_{GPO} = \frac{1}{2\pi}\frac{d\phi_{CEO}}{dt}\,,\ \nu_{CEO} = \left|\nu_{GPO} - k\,f_{rep}\right|\,, \tag{1}$$

where k is an integer chosen to yield the smallest possible value of ν_{CEO}. As illustrated in Fig. 2, ν_{CEO} is defined as the frequency of the line closest to $\nu = 0$ when the comb spectrum is extrapolated to lower frequencies. A non-vanishing value of ν_{GPO} can be attributed to the difference in phase and group delays of the laser cavity. On a single pass through a 2-mm Ti:sapphire crystal, the group is typically delayed 140 fs relative to the phase front. This delay corresponds to more than 50 optical cycles and leads to a typical value of $\nu_{GPO} = 10$ GHz for a KLM laser with $f_{rep} = 100$ MHz. Assuming typical thermal and vibrational fluctuations reported for similar lasers, we expect the resulting frequency fluctuations to be small enough for an unambiguous choice of k. In the following, we will discuss the stabilization and measurement schemes in terms of ν_{CEO}.

Signal characterization by ν_{CEO} is not restricted to bandwidth-limited pulses if the phase relation between adjacent spectral lines is sufficiently stable. In other words, the periodicity of the temporal features of the laser output (amplitude, phase or any combination of both) must be sufficiently high. For a practical ν_{CEO} measurement, this is achieved if the rms amplitude modulation index of an individual mode is small compared to unity and its power is sufficient for unambiguous phase-tracking [8].

The absolute frequency of the mth spectral line of the comb, $\nu(m)$, can be determined from the order number m and the frequency difference of two adjacent modes, f_{rep}, and ν_{CEO}:

$$\nu(m) = \nu_{CEO} + m\,f_{rep}\,. \tag{2}$$

Whereas both the direct detection of f_{rep} with an rf counter and the measurement of m with a wave-meter are straightforward, the determination of ν_{CEO} remains the challenge for absolute frequency measurement and carrier-envelope phase stabilization.

A previous measurement of ν_{CEO} employed an interferometric method [9]. It relies on a cross-correlation of two successive laser pulses in a nonlinear optical crystal. The ν_{CEO} is determined by detecting the second harmonic of the interferometer output signal while the path delay is varied. When $\nu_{CEO} = 0$, subsequent pulses have an identical carrier-envelope relationship and their interferometric cross-correlation yields a function similar to an interferometric auto-correlation trace [10]. In this case, the cross-correlation is symmetric with respect to the roundtrip time $T_R = 1/f_{rep}$. When $\nu_{CEO} \neq 0$, however, the maximum fringe appears at $T_R + \Delta T_{CEO}$ with

$$\Delta T_{CEO} = \frac{\nu_{CEO}}{\nu_0\,f_{rep}}\,, \tag{3}$$

where ν_0 is the optical carrier frequency.

With this method, the interferometer directly measures the frequency of a signal but not its phase. Any attempt to deduce ϕ_{CEO} from ν_{CEO} by integration will most likely fail because the measured value of ν_{CEO} may contain the integration of various unwanted offsets. For example, the difference between the group and phase delay of air in an open interferometer leads to an offset of 1.3 GHz in measuring ν_{CEO}. When the magnitude of this offset is compared with the repetition frequency of a typical KLM laser, $f_{rep} \approx 100$ MHz, it is clear that phase-coherent methods are indispensable for precise ϕ_{CEO} control.

1 Phase-coherent measurement of ν_{CEO} and ϕ_{CEO}

We propose several new schemes for the *phase-coherent* determination of ν_{CEO} that make use of nonlinear optical processes including second-harmonic generation (SHG), sum-frequency generation (SFG), and difference-frequency generation (DFG). The instantaneous phase angles of the generated signals directly reflect ϕ_{CEO} when additional phase offsets due to time delays of the analog signal processing path are taken into account. These offsets can be determined independently and are nearly constant. ϕ_{CEO} can be held at a constant value with the help of a servo loop requiring a control input at the laser to modify the difference between group and phase delay of the laser resonator. This can be accomplished by tilting the reflector behind the group-velocity-dispersion compensating prism set-up [11, 12] or by laterally shifting intra-cavity wedged plates.

The general idea of our schemes is to generate mixing products, via nonlinear processes, from different parts of the comb and/or output signals of additional lasers whose frequencies contain $p\nu_{CEO}$, with $p \neq 1$. The frequencies of the comb modes, however, contain a unity of ν_{CEO}; see Fig. 2. Consequently, heterodyning the mixing product and nearby comb modes results in a beat note oscillating at a $p-1$ multiple of ν_{CEO}.

The complexity of the set-up required to establish a coherent link between the modulation and the carrier frequencies depends on the phase-coherent frequency range of the comb. A scheme with a wider relative comb width, $\Delta\nu/\nu = 2(\nu_{high} - \nu_{low})/(\nu_{high} + \nu_{low})$, requires fewer intermediate oscillators and mixing steps to determine ν_{CEO}.

1.1 Direct SHG or DFG

The simplest method for the measurement of ν_{CEO} requires *one nonlinear optical process*. This method beats the second harmonic of the mth mode from the low-frequency wing of the comb with the $2m$th mode from the high-frequency wing. For any mode within the phase-matching range of the SHG

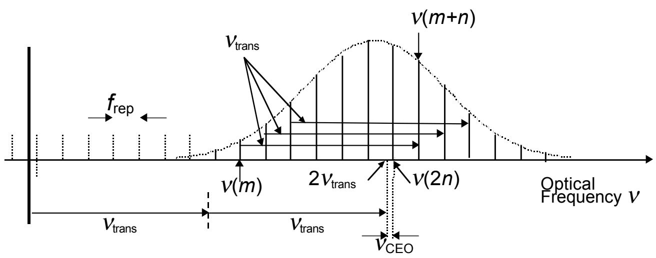

Fig. 3. Proposed ν_{CEO} measurement scheme of Sect. 1.2 using multi-line sum-frequency generation

crystal,

$$2\nu(m)-\nu(2m)=(2m\,f_{\mathrm{rep}}+2\nu_{\mathrm{CEO}})-(2m\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}})\\ =\nu_{\mathrm{CEO}}\,. \tag{4}$$

In this scheme, many modes may contribute to the measurement signal, thereby improving the S/N ratio. However, this scheme requires very wide frequency combs covering one octave (a ratio of the high and low frequency of 2:1 or $\Delta\nu/\nu=0.67$), and the power in the extreme parts of the comb spectrum is usually very low.

The same information can be obtained by DFG between modes from the opposite ends of the comb and the subsequent beating of this mixing product with modes from the low-frequency wing. This alternative may be preferred if the quantum efficiency of the photo-detector is higher in this spectral region.

The SHG scheme can be improved by using one additional oscillator. When the emission field of the auxiliary oscillator is phase-locked to the mth mode, $\nu(m)$, equation 4 still applies and although only one comb line is used, the efficiency of the nonlinear optical process can be much higher when a strong single-frequency auxiliary laser is used.

1.2 Frequency-doubled transfer oscillator

The required span of the comb can be reduced if *two nonlinear-optical processes* (SFG and SHG) and one additional transfer oscillator at ν_{trans} are combined, see Fig. 3. The fundamental at frequency ν_{trans} is used for a collective up-conversion of modes from the low-frequency to the high-frequency end of the comb. If $\nu_{\mathrm{trans}}=n\,f_{\mathrm{rep}}$ then $\nu(m+n+i)=\nu(m+i)+\nu_{\mathrm{trans}}$ for all i leading to modes within the phase-matching range of the SFG. With active control of ν_{trans}, the frequency shift can be forced to an integer number of modes n by tuning the frequency of the beat note between the modes of the comb-like SFG signal and the comb modes in the vicinity of $\nu(m+n)$ to zero. Simultaneously, the second harmonic at $2\nu_{\mathrm{trans}}$ is compared with the frequency of the nearest comb mode $\nu(2n)=2n\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}}$,

$$\nu(2n)-2\nu_{\mathrm{trans}}=(2n\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}})-2n\,f_{\mathrm{rep}}=\nu_{\mathrm{CEO}}\,. \tag{5}$$

As in the direct SHG process, all modes within the phase-matching range of this process contribute to the measured signal. In principle, ν_{trans} can be chosen arbitrarily but must not exceed the frequency span of the comb. The required relative comb width for this method ranges from 0.5 with $2\nu_{\mathrm{trans}}$ chosen at comb center to 0.4 with $2\nu_{\mathrm{trans}}$ chosen at the low-frequency wing of the comb.

1.3 Frequency-tripled transfer oscillator

Given sufficient power, the high mixing efficiency of a combination of powerful laser-diode-pumped solid-state lasers and a quasi-phase-matched nonlinear crystal allows third-harmonic generation (THG). Because THG is usually accomplished by SHG and sum frequency mixing of the second harmonic with the fundamental, *three nonlinear optical processes* are required in total. As in the frequency doubling scheme, the fundamental is used to up-shift modes from the low-frequency wing of the spectrum and the third harmonic is compared with the nearest comb mode at $\nu(3n)=3n\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}}$. This leads to

$$\nu(3n)-3\nu_{\mathrm{trans}}=(3n\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}})-(3n\,f_{\mathrm{rep}})=\nu_{\mathrm{CEO}}\,. \tag{6}$$

A relative comb width between 0.33 at the comb center and 0.29 at the low-frequency wing makes this scheme's comb width significantly smaller than that of the other schemes presented so far. Depending on the SHG efficiency, it might be necessary to employ an additional oscillator phase-locked to the second harmonic at $2\nu_{\mathrm{trans}}$ in order to obtain sufficiently strong SFG signals at $\nu_{\mathrm{trans}}+2\nu_{\mathrm{trans}}$.

1.4 SHG and THG of auxiliary oscillators

By using two additional lasers with frequencies ν_{a} and ν_{b}, the second and third harmonics of these two lasers can be phase-locked at $3\nu_{\mathrm{a}}=2\nu_{\mathrm{b}}$ with a servo loop. In a subsequent step, ν_{a} is phase-locked to the nearest mode $\nu(2m)$,

$$\nu_{\mathrm{a}}=2m\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}}\,, \tag{7}$$

and the beat note between ν_{b} and its nearest comb mode at $\nu(3m)$ provides the desired information about ν_{CEO},

$$\nu_{\mathrm{b}}-\nu(3m)=\frac{3}{2}(2m\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}})-(3m\,f_{\mathrm{rep}}+\nu_{\mathrm{CEO}})\\ =\frac{\nu_{\mathrm{CEO}}}{2} \tag{8}$$

This scheme has the same efficiency constraints as the frequency doubling and tripling schemes and, again, it might be necessary to employ a third oscillator for the THG process.

1.5 Frequency interval bisection

Frequency interval bisection [13] can be employed to divide the required comb width. The simplest scheme of this type uses a one-octave interval divider stage that generates the frequency ν_{a} at the midpoint of ν_{b} and $2\nu_{\mathrm{b}}$. This can be done by phase-locking the second harmonic of ν_{a} to the sum frequency of ν_{b} and $2\nu_{\mathrm{b}}$ to yield

$$\frac{\nu_{\mathrm{a}}}{2\nu_{\mathrm{b}}}=\frac{3\nu_{\mathrm{b}}/2}{2\nu_{\mathrm{b}}}=\frac{3}{4}\,. \tag{9}$$

In a way similar to the SHG and THG methods in auxiliary oscillators, ν_a is phase-locked to the nearest line, $\nu_a = 3m f_{rep} + \nu_{CEO}$, and the beat note between $2\nu_b$ and its nearest comb line at $\nu(4m)$ provides the desired information, namely,

$$2\nu_b - \nu(4m) = \frac{4}{3}(3m f_{rep} + \nu_{CEO}) - (4m f_{rep} + \nu_{CEO}) = \frac{\nu_{CEO}}{3} . \tag{10}$$

Yet again, an additional laser at $2\nu_b$ might be needed to obtain sufficiently strong SFG signals.

1.6 Comparison of different schemes

Table 1 summarizes the number of necessary auxiliary oscillators, the number of nonlinear steps and the required comb width of the proposed schemes. Excluding the direct SHG or DFG schemes, with their very demanding $\Delta\nu/\nu = 0.67$, all schemes can be potentially implemented with state-of-the-art KLM Ti:sapphire lasers.

For a successful implementation of any of the schemes proposed here, it is mandatory that a cycle-slip-free phase-lock between different parts of the spectrum be maintained. This condition is equivalent to rms phase jitters of $\sigma_\phi \approx 0.3$ rad [8], corresponding to CEO timing jitters in the range from 0.1 fs to 0.4 fs, depending on the scheme employed. In the following, we will explore the feasibility of the phase-locks, using experimental data from our laser.

2 Experiments with a sub-6-fs Ti:sapphire laser

The selection of the most appropriate scheme for a given laser system depends on the width and the shape of the laser's emission spectrum. We performed experiments using a SESAM-assisted Kerr-lens mode-locked (KLM) Ti:sapphire laser [1], with an output power of ≈ 300 mW at 8 W of Ar^+-laser pump power, capable of producing sub-6-fs pulses [14]. In the experiments described here, we used slightly longer pulses. In our laser, self-phase-modulation inside the Ti:sapphire crystal causes spectral broadening beyond the gain bandwidth. Because of the strongly reduced reflectivity of the mirrors in the extreme parts of the spectrum, this light is immediately coupled out and not spatially filtered by multiple passes through the cavity. Consequently, the beam quality may deteriorate in the extreme spectral wings of the comb. We measured the power spectrum with a 30-cm double monochromator in additive configuration and determined the wavelength-dependent sensitivity of the monochromator–detector combination with a calibrated white-light source, see Fig. 4. Measurement of the diffraction-limited parts of the beam was ensured by using the spatial filtering of a 5-μm-diameter pinhole.

Table 1. Comparison of the two schemes in terms of additional oscillators, nonlinear conversion steps, and required frequency ranges

Scheme	Eq.	additional oscillators	nonlinear conv. steps	$\nu_{low}:\nu_{high}$	$\Delta\nu/\nu$
1.1 direct SHG/DFG	(4)	0/1	1	1 : 2	0.67
1.2 doubled transfer osc.	(5)	1	2	2 : 3	0.4..0.5
1.3 tripled transfer osc.	(6)	1	3	3 : 4	0.29...0.33
1.4 aux. oscillators	(8)	2/3	3	2 : 3	0.4
1.5 interval bisection	(10)	2/3	3	3 : 4	0.29

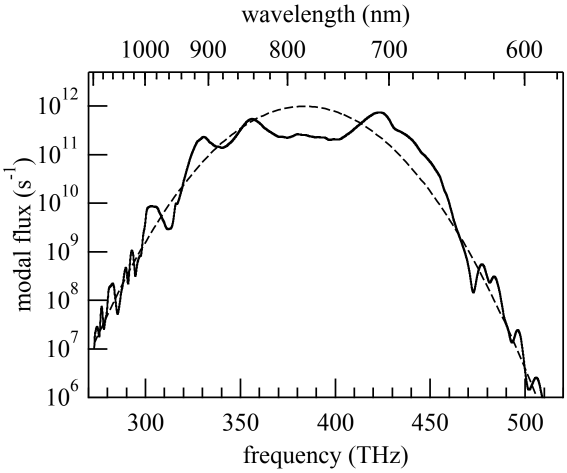

Fig. 4. Measured spectral flux density of the KLM Ti:sapphire laser under diffraction limited conditions. The spectrum is calibrated in units of photons per second and mode. TEM_{00} power is 100 mW. Solid line: measured spectrum. The Fourier transform of this spectrum has a width of 6.6 fs. The minima in the extreme spectral wings correspond to resonances of the mirrors used inside the laser. Dashed line: Gaussian fit with 55 THz FWHM, corresponding to an 8-fs duration of a bandwidth-limited pulse

Selection of the most appropriate scheme for ϕ_{CEO} control also requires consideration of the minimum photon flux per mode. Assuming a detection bandwidth of 100 kHz, 10^5 photons/s are needed for a shot-noise-limited S/N ratio of 1. To avoid cycle slips, the number of detected photons must be at least 100 times this value [8]. Imperfect mode matching, limited quantum efficiency, and other losses may account for an additional factor of 100, which leads to a conservative requirement of 10^9 photons/s per mode.

The measured spectrum in Fig. 4 indicates that the spectral range from 650 to 1000 nm, corresponding to a width of more than 160 THz, satisfies this criterion. This gives $\Delta\nu/\nu = 0.42$. For most of the wavelength range, the spectrum is fitted by a Gaussian with a 55-THz FWHM, corresponding to a pulse duration of 8 fs whereas the transform limit of the diffraction-limited part of the spectrum yields 6.6 fs.

Taking into account the availability of additional lasers at specific frequencies, the schemes discussed in Sects. 1.2 and 1.5 appear to be the most promising candidates for the CEO phase control of our mode-locked laser. However, the frequency-doubled transfer oscillator (Sect. 1.2) requires a comb span of about 150 THz. Laser diodes in this 2 μm wavelength range are not readily available, and provide only small output power at large emission linewidth. The power in the spectral wings is insufficient to bridge larger frequency ranges in the 200-THz range, where convenient powerful lasers are readily available (1.5-μm Er fiber lasers). Thus, of these two schemes, we can more easily meet the requirements of frequency interval bisection. If we choose the strong 1.338-μm Nd:YAG line as the fundamental of the octave-interval, we get the second harmonic at 669 nm and the mid-point frequency at 892 nm. This interval of 669 nm to 892 nm would then have to be bridged by the comb; Fig. 4 shows that our laser provides sufficient power in this wavelength range.

A further advantage of this scheme is that several mW of the power required at 669 nm can be directly generated by SHG from the fundamental, thus eliminating the need for an additional laser. The field at 895 nm can be generated by narrow linewidth DBR laser diodes commercially available for spectroscopy of the cesium D_1 transition. Furthermore, all nonlinear processes can be carried out with highly efficient mixers, including periodically poled KTP for SHG of 1338 nm and non-critically phase-matched $KNbO_3$ for SHG of 892 nm and SFG of 1338 nm and 669 nm.

To investigate the FM noise properties of the KLM laser modes and to show the feasibility of ϕ_{CEO} control, we measured the beat notes of the KLM laser with two independent cw lasers. Two extended cavity laser diodes (ECLD), emitting at 658 nm and 812 nm, were used as local oscillators in this initial experiment. Although these are not the exact wavelengths required for frequency interval bisection, it is reasonable to suggest that the lock at 669 nm will work if the beat at 658 nm is sufficiently strong and coherent. Additionally, because the diffraction-limited single line power is similar at 812 nm and 892 nm, a measurement of the beat at 812 nm indicates the feasibility of the lock at 892 nm.

We measured both beat notes with an RF spectrum analyzer. The Ti:sapphire laser output was attenuated and spectrally filtered in order to avoid unwanted nonlinear effects like self-phase-modulation. We used two interference filters with 20 nm wide pass-bands, centered at about 660 nm and 820 nm, respectively. The ECLD output signal was also attenuated to reduce unwanted optical feedback. Both attenuated signals were coupled into a 5 m piece of single mode fiber for spatial filtering and detected with a fast (∼ 200 MHz) Si PIN photo-diode. We measured a 16-dB S/N ratio (BW = 100 kHz) of the beat note, as shown in Fig. 5a. Because the heterodyne detection scheme is insensitive to the sign of the beat note frequency, two line clusters appear at frequencies symmetric with respect to $f_{rep}/2$.

A calculation of the theoretical S/N, derived from the average Ti:sapphire laser power reaching the photodiode, indicates that there is some potential for improvement in the experimental value. The $\sim 10^5$ modes of the Ti:sapphire laser passing the interference filter correspond to a total dc photo current of $I_{TOT} = 50\,\mu A$ or $I_{SM} = 5\times10^{-10}$ A per single mode. Taking into account the measured value of $3\,\mu A$ for the photo-current from the ECLD acting as local oscillator (LO), the preamplifier input noise density of $(i_{AMP})^2 = 8\times10^{-24}\,A^2/Hz$, and assuming that AM noise is negligible, one expects a S/N ratio of

$$\frac{S}{N} = \frac{1}{\Delta f}\frac{I_{SM}I_{LO}}{2e(I_{TOT}+I_{LO})+(i_{AMP})^2} \cong \frac{6\times10^7\,\mathrm{Hz}}{\Delta f}\,, \qquad (11)$$

where $2e(I_{TOT}+I_{LO})$ accounts for the total shot noise, Δf is the detection bandwidth, and e is the electron charge. From this number we calculate S/N≈ 28 dB for a Δf of 100 kHz, as opposed to the experimental value of 16 dB found in Fig. 5a. Part of this deviation can be attributed to limitations of the observed S/N by the fast linewidth[1] [15]. Furthermore, the experimental value of 16 dB can be easily improved by 15 dB if the LO photocurrent is increased to $I_{LO} > 50\,\mu A$ by using an isolator rather than an attenuator to prevent back reflections into the laser diode. In that case, the shot noise of the LO becomes the dominant noise contribution and the S/N ratio for a given signal power cannot be further improved.

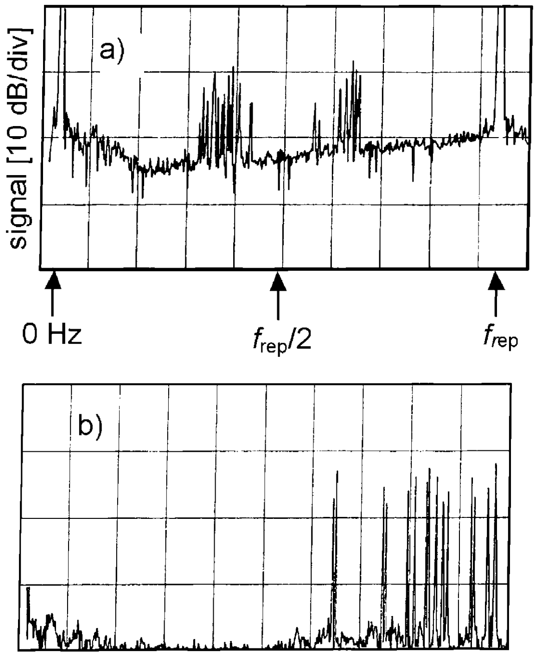

Fig. 5a,b. Measured beat notes between Ti:sapphire Laser and ECLD emission. **a** $\lambda = 658$ nm, 10 dB/div, 10 MHz/div, Res. BW = 100 kHz, I_{PD} = 50 µA (Ti:sapphire), 3 µA (LD) **b** $\lambda = 812$ nm, 10 dB/div, 2 MHz/div, Res. BW = 30 kHz, I_{PD} = 20 µA (Ti:sapphire), 60 µA (LD), sweep time = 10 ms/div

Both beat notes showed frequency fluctuations of about 100 MHz on a time scale of a few hundred milliseconds. These are due to air pressure variations inside the open Ti:sapphire laser cavity and due to thermally induced changes of the SESAM reflection phase. In addition, we observed frequency fluctuations of about 10 MHz on a millisecond time scale, due to acoustic length variations of the Ti:sapphire laser and ECLD cavities. Suppression of the slow variations is possible with the sufficiently fast control elements available, thus we are mainly concerned with the high-frequency noise contributions broadening the beat note. Such noise components ultimately determine the required detection bandwidth and consequently the minimum single-mode power required for unambiguous ϕ_{CEO} control.

Signals with large frequency fluctuations appearing exclusively in the low-frequency range can be reasonably characterized by the width of their instantaneous spectrum, commonly referred to as the fast linewidth. Under our conditions, an upper boundary for the fast linewidth can be estimated from the following procedure. First, the beat note is analyzed with a very large detection bandwidth to ensure that the full beat note power is detected. The next step is to reduce the

[1] Assuming a phase-lock loop which phase-tracks the signal, the fast linewidth can be understood as the unity gain frequency of the loop which is required to yield an rms value of the residual error signal of less than 1 rad. This situation is equivalent to a tracking filter, which tightly follows any low-frequency fluctuations of the input signal. Then the fast linewidth corresponds to the minimum filter bandwidth required to yield a power transmission of > 0.5.

spectrum analyzer bandwidth. As a result of low-frequency fluctuations, the beat note is detected several times during one spectrum analyzer sweep and each spectral line appears to be split into several lines (Fig. 5). The height of these lines will be reduced compared to the reference level measured with the large bandwidth if both the filter and the signal frequencies coincide for a time period shorter than the filter storage time. As long as some lines can still be detected at the reference level, one can conclude that the fast linewidth of the beat note is smaller than the current filter bandwidth.

Following this procedure, we estimated upper boundaries of the fast linewidths to be 30 kHz and 100 kHz for the beat notes at 812 nm and 658 nm, respectively. Independent measurements show that the fast emission linewidths of the ECLDs themselves are on the same order. Thus, we conclude that the fast linewidth of a single Ti:sapphire laser emission line is smaller than 100 kHz in the frequency range of interest and that our 10^9 photons/s criterion is appropriate.

3 Conclusion

Several novel schemes providing a phase-coherent link between the modulation frequency and the carrier frequency of a short laser pulse have been presented. These methods are capable of measuring the instantaneous CEO phase of a periodic pulse train emitted by a mode-locked laser. Initial experiments show that a comb span of 87 THz can be bridged with a phase-coherent link. The Ti:sapphire laser output spectrum measured under diffraction-limited conditions clearly indicates that the useful span of the frequency comb exceeds 160 THz, which allows a simple frequency interval bisection scheme to be used to measure and stabilize ϕ_{CEO}. For this scheme, only one KLM Ti:sapphire laser and two commercially available auxiliary cw lasers are required. CEO timing jitters in the 0.1-fs range can be anticipated for the proposed stabilization schemes.

Together with a measurement of the pulse repetition frequency and the determination of the mode order, a direct and phase-coherent link from any frequency covered by the comb spectrum to a primary microwave standard is possible. Also, the stabilization of the CEO phase opens an avenue in ultrashort pulse generation for high-harmonic generation and attosecond pulse generation.

Acknowledgements. The authors gratefully acknowledge financial support by ETH Zürich under grant no. 03159/41.

References

1. D.H. Sutter, G. Steinmeyer, L. Gallmann, N. Matuschek, F. Morier-Genoud, U. Keller, V. Scheuer, G. Angelow, T. Tschudi: Opt. Lett. **24**, 631 (1999)
2. U. Morgner, F.X. Kärtner, S.H. Cho, Y. Chen, H.A. Haus, J.G. Fujimoto, E.P. Ippen, V. Scheuer, G. Angelow, T. Tschudi: Opt. Lett. **24**, 411 (1999); Opt. Lett. **24**, 920 (1999)
3. A.d. Bohan, P. Antoine, D.B. Milosevic, B. Piraux: Phys. Rev. Lett. **81**, 1837 (1998)
4. M. Ivanov, P.B. Corkum, T. Zuo, A. Bandrauk: Phys. Rev. Lett. **74**, 2933 (1995)
5. I.P. Christov, M.M. Murnane, H.C. Kapteyn: Phys. Rev. Lett. **78**, 1251 (1997)
6. J. Ye, L.-S. Ma, T. Daly, J.L. Hall: Opt. Lett. **22**, 301 (1997)
7. Th. Udem, J. Reichert, R. Holzwarth, T.W. Hänsch: Optics Lett. **24**, 881 (1999)
8. H.R. Telle: In *Frequency control of semiconductor lasers*, ed. by M. Ohtsu (Wiley, New York 1996) p. 137
9. L. Xu, C. Spielmann, A. Poppe, T. Brabec, F. Krausz, T.W. Hänsch: Opt. Lett. **21**, 2008 (1996)
10. J.-C. Diels, W. Rudolph: *Ultrashort Laser Pulse Phenomena* (Academic Press, San Diego 1996)
11. Th. Udem, J. Reichert, R. Holzwarth, T.W. Hänsch: Phys. Rev. Lett. **82**, 3568 (1999)
12. K.F. Kwong, D. Yankelevich, K.C. Chu, J.P. Heritage, A. Dienes: Opt. Lett. **18**, 558 (1993)
13. H.R. Telle, D. Meschede, T.W. Hänsch: Opt. Lett. **15**, 532 (1990)
14. L. Gallmann, D.H. Sutter, N. Matuschek, G. Steinmeyer, U. Keller, C. Iaconis, I.A. Walmsley: Opt. Lett. **24**, 1314 (1999)
15. D. Halford: In *Proceedings of the Frequency Standards and Metrology Seminar* (Laval University, Quebec, Canada 1971) p. 431